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Frontiers in Artificial Intelligence Algorithm Optimization: Fermatean Fuzzy Deep Neural Networks for Uncertainty-Aware Decision-Making

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Abstract: With the rapid development of Artificial Intelligence (AI) technology, contemporary AI decision-making systems face significant challenges when dealing with high levels of uncertainty, imprecise data, and complex decision-making environments. Traditional deep learning models often struggle to maintain performance under such ambiguous conditions. Fermatean Fuzzy Theory (FFT), which utilizes advanced fuzzy numbers to comprehensively describe uncertainty, provides AI systems with enhanced flexibility and robustness. This is especially critical in fields such as multi-criteria decision-making, compromise programming, and reinforcement learning. To further improve the capability of uncertainty modeling, this paper proposes a novel Fermatean Fuzzy Deep Neural Network (FF-DNN) framework by systematically integrating Fermatean fuzzy theory into deep learning architectures. This innovative framework enables the rigorous fuzzification of input data, network weights, and activation functions, thereby significantly enhancing the overall robustness, generalization, and adaptability of neural networks operating in highly uncertain environments. From the perspective of artificial intelligence algorithm optimization, this study deeply explores the synergistic integration of fuzzy theory and deep learning for uncertainty-aware decision-making. Furthermore, this paper comprehensively examines the practical application of Fermatean Fuzzy Theory in AI decision-making, particularly highlighting its distinct advantages and inherent challenges in handling uncertainty and fuzziness. Finally, the study validates the effectiveness and superiority of the proposed FF-DNN framework through rigorous theoretical analysis and extensive case study discussions, demonstrating its potential to revolutionize complex decision support systems.

Keywords: fuzzy theory; deep learning; artificial intelligence; uncertainty modeling; decision support; algorithm optimization

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1. Introduction

Artificial Intelligence (AI) decision-making is widely applied in fields such as healthcare, finance, and autonomous driving, where it addresses various complex and uncertain decision-making problems. These decision problems often involve multiple criteria and significant uncertainty [1]. For instance, autonomous driving systems need to make decisions while considering road conditions, traffic situations, passenger safety, and other uncertain factors, while financial decisions must account for market volatility, investment returns, and risks.

Traditional decision-making methods, such as weighted average methods and linear programming, typically assume that information is precise and deterministic. This assumption is insufficient when addressing complex, uncertain real-world problems [2]. Fermatean Fuzzy Theory (FFT), through the introduction of fuzzy numbers and confidence assessments, effectively addresses uncertainty and provides more flexible and reliable support for AI decision-making systems.

Recent advances in artificial intelligence algorithm optimization emphasize the importance of integrating uncertainty modeling into deep learning frameworks [1].

Research Objectives and Contributions: The primary aim of this paper is to explore how Fermatean Fuzzy Theory can effectively handle uncertainty in AI decision-making, particularly in multi-criteria decision-making, compromise programming, and reinforcement learning applications [3]. The contributions of this paper are as follows:

1. A detailed definition and mathematical derivation of Fermatean fuzzy numbers, highlighting their advantages over traditional fuzzy numbers;
2. An in-depth discussion of Fermatean Fuzzy Theory's application in AI decision-making, particularly in dynamic and variable decision environments;
3. Empirical validation of the robustness and adaptability of Fermatean Fuzzy Theory in real-world applications.

2. Fermatean Fuzzy Theory: Foundations and Uncertainty Modeling Paradigm

2.1. Uncertainty in Artificial Intelligence Decision Systems

Modern artificial intelligence systems increasingly operate in environments characterized by ambiguity, incompleteness, and stochastic variability. In domains such as autonomous driving, healthcare analytics, and financial forecasting, data is often noisy, partially observable, and inherently uncertain [4]. Traditional deterministic modeling paradigms, as well as classical probabilistic approaches, face limitations in simultaneously addressing both epistemic uncertainty, which arises from incomplete knowledge, and vagueness, stemming from imprecise boundaries in human reasoning.

Fuzzy set theory has been widely adopted to address vagueness; however, standard fuzzy models are insufficient for representing higher-order uncertainty structures, particularly when membership and non-membership information coexist under constrained conditions. This limitation has driven the development of more expressive fuzzy frameworks capable of encoding richer uncertainty semantics for intelligent decision-making systems [2].

2.2. Evolution from Classical Fuzzy Sets to Fermatean Fuzzy Representation

To enhance uncertainty modeling capability, several extensions of fuzzy set theory have been proposed, including intuitionistic fuzzy sets and Pythagorean fuzzy sets. These models introduce dual representations of membership and non-membership degrees, improving flexibility in uncertainty representation [5].

Fermatean fuzzy theory further extends this progression by introducing a higher-order structural framework that allows a broader admissible space for uncertainty representation [4]. Compared to previous fuzzy frameworks, Fermatean fuzzy representation provides greater expressive freedom while maintaining mathematical consistency in decision modeling.

The core advantage of this framework lies in its ability to simultaneously accommodate stronger degrees of membership and non-membership while preserving a valid uncertainty boundary, making it particularly suitable for complex decision environments with high ambiguity and diverse evidence [6].

2.3. Conceptual Characteristics of Fermatean Fuzzy Information

Fermatean fuzzy information is characterized by a triplet-based representation capturing three essential aspects of uncertainty:

1. Support (membership tendency): representing the degree to which an element satisfies a given property
2. Non-membership tendency: representing the degree to which the element does not satisfy the property
3. Indeterminacy (hesitation or ambiguity): representing unresolved uncertainty due to insufficient or varying evidence

Unlike conventional fuzzy representations, Fermatean fuzzy information explicitly emphasizes the balance between support and non-membership under constrained uncertainty space, allowing more flexible modeling of real-world ambiguity.

This structural design provides a more realistic abstraction of human-like reasoning processes, where decisions are rarely binary or fully certain, but instead constructed under partial belief systems [7].

2.4. Role of Fermatean Fuzzy Theory in Decision Intelligence

From the perspective of decision intelligence, Fermatean fuzzy theory serves as a bridge between human subjective reasoning and machine-driven optimization. It enables the formal representation of uncertain preferences, incomplete judgments, and ambiguous criteria in multi-criteria decision-making systems [8].

In particular, Fermatean fuzzy modeling is highly effective in scenarios where:

1. Decision criteria are diverse or difficult to compare quantitatively
2. Evaluation information is incomplete or linguistically expressed
3. System states evolve dynamically under uncertainty
4. Human expert knowledge must be integrated with data-driven inference

These characteristics make Fermatean fuzzy theory a suitable foundational tool for next-generation AI systems that require robust uncertainty-aware reasoning capabilities [9].

2.5. Positioning Fermatean Fuzzy Theory in Deep Learning Context

Traditional fuzzy systems are primarily designed for rule-based reasoning, whereas modern AI systems necessitate integration with data-driven architectures such as deep neural networks [10]. Conventional neural networks, however, operate on deterministic numerical representations, which inherently reduce uncertainty during forward propagation.

Fermatean fuzzy theory introduces a structured mechanism to maintain uncertainty throughout computational layers by embedding fuzzy representations into inputs, parameters, and intermediate transformations. This approach enables uncertainty to propagate rather than diminish, offering a more accurate representation of real-world information dynamics within learning systems [11].

This conceptual alignment between fuzzy uncertainty modeling and deep learning architectures establishes the theoretical basis for the proposed Fermatean Fuzzy Deep Neural Network (FF-DNN).

2.6. Research Gap and Motivation

Recent advancements in fuzzy-based machine learning models have highlighted several critical limitations [12].

Current fuzzy neural systems predominantly utilize low-order uncertainty representations, which restrict their expressive capacity in complex scenarios.

Hybrid models often struggle to consistently propagate uncertainty through deep architectures, leading to a partial loss of interpretability [6].

Unified frameworks that seamlessly integrate fuzzy uncertainty modeling with modern deep learning optimization structures in a mathematically consistent manner remain scarce [12].

This study introduces a Fermatean Fuzzy Deep Neural Network (FF-DNN) framework, designed to incorporate higher-order fuzzy uncertainty representation within deep learning architectures. The objective is to improve both robustness and interpretability in AI decision-making under uncertain conditions [9].

2.7. Conceptual Case: Uncertainty-Aware Autonomous Driving Decision Scenario

To further illustrate the role of Fermatean fuzzy theory in complex decision environments, consider a representative autonomous driving scenario under challenging weather conditions [3].

In such environments, an autonomous vehicle must continuously interpret sensor inputs from multiple sources, including cameras, LiDAR, and radar [8, 11]. However, these inputs are often affected by environmental factors such as fog, rain, partial occlusion, and lighting variations.

From a traditional deep learning perspective, such factors are typically treated as noise and implicitly suppressed during feature extraction [1]. However, this may lead to the loss of critical uncertainty information, especially in safety-critical decision-making tasks.

Within the Fermatean fuzzy framework, sensor reliability, object detection confidence, and environmental clarity can be represented as Fermatean fuzzy information, where:

1. .the membership degree reflects the confidence that an object or obstacle is correctly detected,
2. .the non-membership degree reflects the confidence that the detection is unreliable or incorrect,
3. .the hesitation degree captures ambiguity caused by incomplete or inconsistent sensor evidence.

Under this representation, the decision-making system does not eliminate uncertainty but explicitly preserves and propagates it through subsequent reasoning layers.

For instance, when the vehicle encounters a partially occluded pedestrian, the system may assign:

1. .high membership degree to "pedestrian presence" from radar signals,
2. .moderate non-membership degree from camera ambiguity,
3. .significant hesitation due to inconsistent multi-sensor fusion outputs.

This uncertainty structure allows the decision system to avoid premature deterministic conclusions and instead propagate graded confidence information into the downstream decision module [6].

Consequently, braking, steering, or acceleration decisions are not based solely on deterministic classification outputs, but on uncertainty-aware reasoning that reflects both confidence and risk levels [3].

This illustrates how Fermatean fuzzy theory enhances decision robustness in safety-critical AI systems by explicitly modeling uncertainty rather than suppressing it [9].

3. Mathematical Foundations of Fermatean Fuzzy Deep Neural Networks

3.1. Fermatean Fuzzy Set (FFS)

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe of discourse [3]. A Fermatean fuzzy set F in X is defined as follows:

$$F = \{\langle x, m_F(x), n_F(x)\rangle \ \ | x \in X\}$$

Here, $m_F(x) \in [0,1]$ and $n_F(x) \in [0,1]$ represent the membership and non-membership degrees, which satisfy the following conditions:

$$m_F^3(x) + n_F^3(x) \leq 1$$

The hesitation degree is expressed as:

$$\pi_F(x) = (1 - m_F^3(x) - n_F^3(x))^{1/3}$$

3.2. Fermatean Fuzzy Number (FFN)

A Fermatean fuzzy number is characterized as:

$$\tilde{\alpha} = (m_\alpha, n_\alpha),$$

where the parameters are defined as:

$$0 \leq m_\alpha, n_\alpha \leq 1, \qquad m_\alpha^3 + n_\alpha^3 \leq 1.$$

The hesitation degree is expressed as:

$$\pi_\alpha = (1 - m_\alpha^3 - n_\alpha^3)^{1/3}$$

3.3. Score and Accuracy Functions

For a Fermatean fuzzy number $\tilde{\alpha} = (m_\alpha, n_\alpha)$, the score function is defined as a measure of the relative preference of an alternative [10].

$$\mathrm{S}(\tilde{\alpha}) = m_\alpha^3 - n_\alpha^3, \mathrm{S}(\tilde{\alpha}) \in [-1,1]$$

This function evaluates the comparative desirability of different options [1, 5].

The accuracy function is defined as a metric representing the reliability of the evaluation [6].

$\mathrm{H}(\tilde{\alpha}) = m_\alpha^3 - n_\alpha^3, \mathrm{H}(\tilde{\alpha}) \in [-1,1]$

It quantifies the dependability of the assessment process.

3.4. Ranking Rule

Let $\tilde{\alpha}_1 = (m_1, n_1)$ and $\tilde{\alpha}_2 = (m_2, n_2)$ be two Fermatean fuzzy numbers. Their ranking is defined as follows:

$\tilde{\alpha}_1 \succ \tilde{\alpha}_2 \quad if \;\; S(\alpha_1) > S(\alpha_2)$

$\tilde{\alpha}_1 \prec \tilde{\alpha}_2 \quad if \;\; S(\alpha_1) < S(\alpha_2)$

If the condition holds:

$S(\alpha_1) = S(\alpha_2)$,

then the ranking is determined accordingly:

$\tilde{\alpha}_1 \succ \tilde{\alpha}_2 \quad if \;\; H(\alpha_1) > H(\alpha_2)$

3.5. Fermatean Fuzzy Weighted Aggregation Operators

Let $\tilde{\alpha}_j = (m_j, n_j)$, $j = 1,2,\cdots,n$ be Fermatean fuzzy numbers with weights ω_j, where:

ωj ≥ 0 and $\sum_{j=1}^{n} \omega_j = 1$.

FFWA refers to Fermatean Fuzzy Weighted Averaging [7].

$FFWA(\tilde{\alpha}_1, \cdots, \tilde{\alpha}_n) = \left(\sqrt[3]{1 - \prod_{j-1}^{n}(1 - m_j^3)^{\omega_j}}, \prod_{j-1}^{n} n_j{}^{\omega_j}\right)$

FFWG refers to Fermatean Fuzzy Weighted Geometric [9].

$FFWG(\tilde{\alpha}_1, \cdots, \tilde{\alpha}_n) = \left(\prod_{j-1}^{n} m_j{}^{\omega_j}, \sqrt[3]{1 - \prod_{j-1}^{n}(1 - n_j^3)^{\omega_j}},\right)$

3.6. Mathematical Foundations of Ff-dnn

Based on the above definitions, the Fermatean Fuzzy Deep Neural Network (FF-DNN) integrates uncertainty into neural computation by representing all components as Fermatean fuzzy numbers.

Input Fuzzification

$\tilde{x}_i = (m_i, n_i),\; m_i^3 + n_i^3 \le 1.$

Weight and Bias Fuzzification

$\tilde{\omega}_{ij} = (m_{ij}^{\omega}, n_{ij}^{\omega}),\; \tilde{b}_{ij} = (m_{ij}^{b}, n_{ij}^{b}),$

Fuzzy Forward Propagation $\tilde{z}_j = (\left[1 - \prod_{i=1}^{n}\left(1 - \left(m_{ij}^{\omega} m_i\right)^3\right)\right]^{\frac{1}{3}}, \prod_{i=1}^{n}(n_{ij}^{\omega} n_i)) \oplus \tilde{b}_j$

The final fuzzy activation output is given by:

$\tilde{h}_j = \phi(\tilde{z}_j)$

where $\phi(\cdot)$ is a Fermatean fuzzy activation function that preserves the condition $m_\alpha^3 + n_\alpha^3 \le 1$..

Output and Decision Representation

$\tilde{y}_j = (m_j, n_j).$

The decision score and confidence are defined as:

$\mathrm{S}(\tilde{y}_j) = m_j^3 - n_j^3,$

$\mathrm{H}(\tilde{y}_j) = m_j^3 + n_j^3,$

The final decision is:

$\mathrm{Decision}_j = (\mathrm{S}(\tilde{y}_j), \mathrm{H}(\tilde{y}_j))$

Figure 1 illustrates the architecture of the proposed Fermatean Fuzzy Deep Neural Network (FF-DNN), showing how fuzzy uncertainty is propagated through the network.

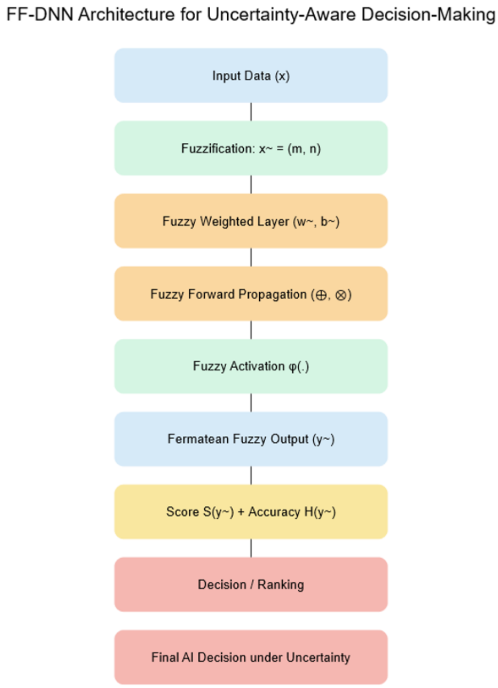

Figure 1. Architecture of the Proposed Ff-dnn Model

The proposed Fermatean Fuzzy Deep Neural Network (FF-DNN) integrates fuzzy uncertainty modeling into each layer of a traditional deep neural network. Input data is first transformed into Fermatean fuzzy representations through membership and non-membership degrees. These fuzzy inputs are then propagated through fuzzy-weighted layers, where both weights and biases are represented as Fermatean fuzzy numbers.

During forward propagation, uncertainty is preserved through fuzzy aggregation mechanisms (FFWA and FFWG operators). The activation function operates in the Fermatean fuzzy domain, ensuring that uncertainty is not diminished during nonlinear transformation.

Finally, the output is expressed as a Fermatean fuzzy number, which is evaluated using score and accuracy functions to obtain the final decision ranking and reliability estimation [10].

4. Application of Fermatean Fuzzy Theory in AI Decision-Making

Fermatean Fuzzy Theory offers a versatile framework for addressing uncertainty in artificial intelligence systems. By moving beyond deterministic models, it allows AI systems to effectively handle ambiguity, incomplete data, and environmental noise, thereby enhancing their robustness and adaptability.

4.1. Multi-Criteria Decision-Making under Uncertainty

In real-world decision problems, multiple criteria must be considered simultaneously [6]. Traditional deterministic methods are often insufficient in addressing uncertainty in evaluation processes.

Fermatean fuzzy modeling enables AI systems to represent evaluation criteria in an uncertainty-aware manner, allowing more flexible analysis among various objectives.

For example, in healthcare decision support systems, treatment selection requires balancing effectiveness, cost, and risk under uncertain clinical conditions. Fermatean fuzzy representation allows these factors to be modeled more realistically, enhancing decision reliability.

4.2. Decision Optimization in Dynamic Environments

Many AI systems operate in dynamic and continuously changing environments, such as transportation systems, financial markets, and resource allocation networks.

Fermatean fuzzy representation enables the modeling of uncertainty in system states and environmental inputs, allowing decision systems to adapt more effectively to real-time changes and incomplete observations [12].

For instance, in intelligent traffic systems, congestion levels, travel time, and risk factors are inherently uncertain and time-dependent [11]. Fermatean fuzzy modeling improves adaptability in such environments.

4.3. Reinforcement Learning under Uncertainty

Reinforcement learning agents function within stochastic and partially observable environments, where uncertainty stems from noisy observations, incomplete state information, and dynamic transitions.

Fermatean fuzzy modeling strengthens reinforcement learning by refining the representation of uncertainty in state data and environmental feedback, thereby facilitating more stable and robust policy development [6].

In autonomous driving scenarios, sensor inputs such as camera and radar signals are frequently impacted by noise and occlusion [2, 8]. Fermatean fuzzy modeling enhances decision-making robustness under such imperfect perception conditions (As shown in Figure 2).

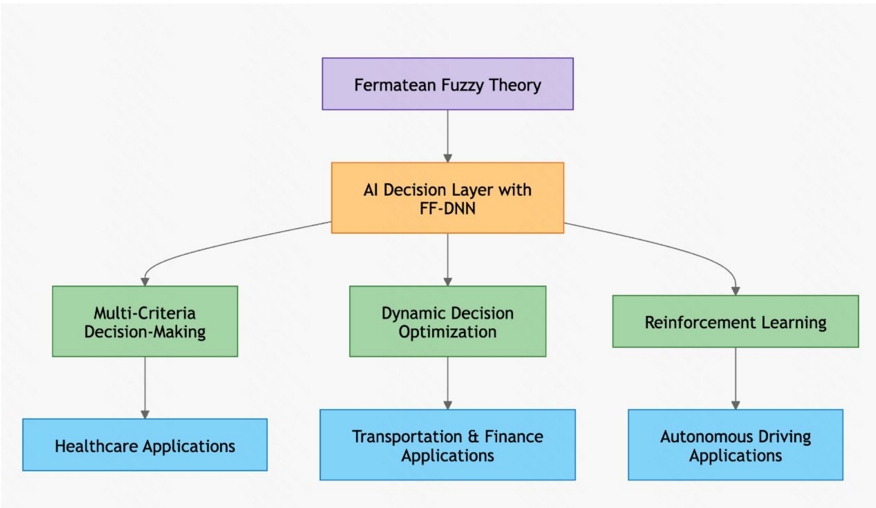

Figure 2. Application Framework of Fermatean Fuzzy Theory in AI Decision-Making under Uncertainty

5. Integration of Deep Learning with Fermatean Fuzzy Theory: Addressing Uncertainty in AI Decision-Making

With the widespread application of deep learning in AI, particularly in complex decision-making tasks, traditional deep neural networks (DNNs) have achieved substantial improvements in accuracy. However, they continue to exhibit limitations in handling uncertainty and fuzzy data [3] (As shown in Figure 3).

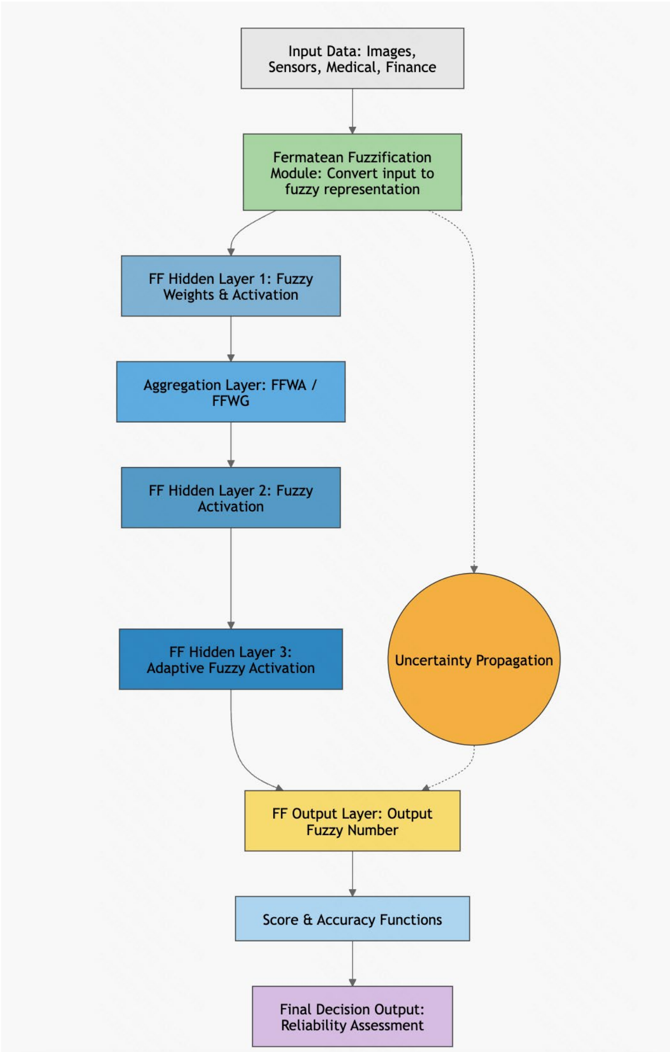

Figure 3. Architecture of Fermatean Fuzzy Deep Neural Network for Uncertainty-Aware Decision-Making

5.1. Fermatean Fuzzy Deep Neural Network (Ff-dnn)

The Fermatean Fuzzy Deep Neural Network (FF-DNN) integrates Fermatean Fuzzy Theory into the deep learning framework. By utilizing fuzzy numbers to represent inputs, weights, and activation functions, this approach enhances the network's ability to manage uncertainty and fuzziness in data while ensuring precision and robustness [5, 9].

5.1.1. Fuzzification of Input Data

Traditional deep learning models typically assume input data to be deterministic. However, real-world applications often involve input data with inherent noise or uncertainty [9]. For instance, in image recognition, variables such as camera angles, lighting conditions, and object occlusion can influence the quality of the input image. The FF-DNN approach addresses this by representing input data using Fermatean fuzzy numbers, which simulate uncertainty across various scenarios, thereby enhancing the network's adaptability to uncertain data.

5.1.2. Fuzzification of Weights and Activation Functions

In conventional deep neural networks, weights and activation functions are treated as fixed values. In FF-DNN, these parameters are modeled using Fermatean fuzzy numbers, which encapsulate the uncertainty range of each parameter. This approach enables the network to dynamically adjust its parameters in response to varying training data and environments, enhancing its ability to handle noisy and uncertain data effectively.

For instance, during training, FF-DNN can refine the network's weights by utilizing the minimum, most likely, and maximum values derived from fuzzy numbers [1]. This mechanism allows the network to identify reliable patterns even when confronted with noise or incomplete data.

5.2. Addressing Uncertainty in AI Decision-Making

Fermatean Fuzzy Theory enables FF-DNN to effectively manage uncertainty in deep learning tasks, offering significant advantages [12].

5.2.1. Enhanced Robustness

Deep learning models often depend on extensive datasets for training, which may include noise or missing values [1, 2]. Representing such data with fuzzy numbers allows FF-DNN to enhance model robustness. This is particularly beneficial in critical applications such as autonomous driving and medical diagnosis, where FF-DNN effectively manages noise or incomplete sensor inputs, thereby minimizing decision errors caused by data uncertainties.

In autonomous driving systems, sensor data from sources like radar and cameras is frequently influenced by external factors, including weather conditions, lighting variations, and traffic dynamics. By employing Fermatean fuzzy numbers, FF-DNN can address these uncertainties, enabling robust decision-making and enhancing both safety and reliability.

5.2.2. Dynamic Adaptation to Environmental Changes

AI decision-making often operates within dynamic environments, such as price fluctuations in financial markets or real-time road conditions in autonomous driving, which are inherently uncertain. FF-DNN enables the dynamic adjustment of the minimum, most likely, and maximum values of fuzzy numbers, allowing the model to adapt to varying decision contexts and enhance decision accuracy.

In autonomous driving, FF-DNN can dynamically modify fuzzy parameters based on real-time sensor inputs, thereby optimizing decision-making processes. Similarly, in financial market prediction, FF-DNN can adjust model outputs in response to market volatility, delivering more precise predictions.

5.2.3. Increased Decision Confidence

In FF-DNN, fuzzy numbers not only represent uncertainty in data but also integrate confidence assessments, effectively quantifying the reliability of each decision. By incorporating confidence evaluation into the model, the system enhances its ability to account for the reliability of uncertain data sources, thereby improving the credibility and precision of its decisions [6].

The decision-making system can dynamically adjust for varying levels of data reliability, ensuring more accurate outcomes. This approach strengthens the model's capacity to handle uncertainty and reinforces the trustworthiness of its recommendations.

For example, in intelligent healthcare decision-making, FF-DNN evaluates confidence levels for each decision by analyzing a patient's historical data alongside its inherent uncertainty. This enables the system to deliver more dependable recommendations to healthcare professionals.

5.3. Case Study: Application in Autonomous Driving Systems

Sensor data in autonomous driving systems, such as those from LiDAR, cameras, and radar, is frequently influenced by external factors like weather conditions, lighting variations, and the occlusion of vehicles or obstacles. These disruptions can result in incomplete or noisy data, which may compromise the accuracy of traditional deep learning models and adversely affect safety and driving performance.

The integration of Fermatean Fuzzy Theory with deep neural networks (FF-DNN) offers a solution to these challenges. For instance, when sensor input images are partially occluded, FF-DNN can model the fuzzy regions within the image and evaluate their usability and reliability by analyzing the minimum, most likely, and maximum values of the fuzzy number. This approach enhances the robustness of autonomous driving systems, minimizing decision errors caused by inconsistencies in sensor data.

5.4. Theoretical Comparison between Ff-dnn and Traditional Deep Neural Networks

To emphasize the advantages of FF-DNN, Table 1 outlines the primary theoretical distinctions between traditional deep neural networks and the proposed Fermatean Fuzzy Deep Neural Network (FF-DNN).

Table 1. Key Differences between Traditional DNN and Ff-dnn

Aspect	Traditional DNN	FF-DNN
Input representation	Deterministic values	Fermatean fuzzy numbers
Weight representation	Fixed parameters	Fuzzy parameter sets
Activation function	Fixed activation function	Fuzzy adaptive activation function
Uncertainty handling	Weak	Strong
Adaptability to dynamic environments	Limited	High

In contrast to traditional deep neural networks, FF-DNN integrates fuzzy representations throughout multiple layers of the learning process, allowing the model to effectively capture uncertainty in both input data and network parameters. This methodology substantially improves robustness in dynamic and noisy environments, making FF-DNN particularly suitable for uncertainty-aware decision-making tasks in complex and variable scenarios.

6. Advantages and Challenges of Fermatean Fuzzy Theory in Handling Uncertainty in AI Decision-Making

6.1. Advantages

Precise Uncertainty Modeling: Fermatean Fuzzy Theory accurately models uncertainty through three-point fuzzy numbers, providing a comprehensive representation of uncertainty compared to traditional models. It offers finer granularity in decision support, making it suitable for complex decision-making environments.

(1).Flexibility in Multi-Criteria Decision-Making: Fermatean fuzzy numbers enable the representation of each decision criterion as a fuzzy number, facilitating scientific trade-offs and compromise decisions in multi-criteria settings [9, 10].

(2).Enhanced Robustness and Adaptability: By incorporating confidence assessment, Fermatean Fuzzy Theory improves the robustness and adaptability of AI systems when handling incomplete, erroneous, or ambiguous information, ensuring reliable decision-making in complex and dynamic environments.

(3).Dynamic Adjustment and Real-Time Decision-Making: As environmental factors evolve, decision objectives and parameters often shift. Fermatean Fuzzy Theory supports dynamic adjustments of objective values and parameters, offering greater flexibility and responsiveness in decision-making, particularly in applications such as autonomous driving and intelligent healthcare.

6.2. Challenges

Computational Complexity: As the scale of decision problems increases, particularly with a growing number of criteria, the computational complexity of Fermatean Fuzzy Theory also increases. This can lead to reduced algorithmic efficiency in large-scale applications.

Solution: Approximation methods or optimization algorithms, such as Particle Swarm Optimization or Genetic Algorithms, can be employed to mitigate the complexity of fuzzy number calculations. Furthermore, parallel and distributed computing techniques can be utilized to enhance computational efficiency.

Dependence on Data Quality: The effectiveness of Fermatean fuzzy numbers is contingent upon the accuracy and completeness of the data. Unreliable or missing data may introduce errors in the construction of fuzzy numbers, thereby impacting decision accuracy [5].

Solution: Data preprocessing techniques, including missing data imputation and noise removal, can be applied to enhance data quality. Additionally, multi-source data fusion methods can improve the reliability and precision of fuzzy number construction [5, 12].

Implementation Complexity: Integrating Fermatean Fuzzy Theory with existing AI algorithms, such as deep learning and reinforcement learning, necessitates substantial model adaptation and adjustments, often involving intricate mathematical modeling and debugging.

Solution: To streamline implementation, specialized tools and frameworks can be developed to assist researchers in integrating Fermatean Fuzzy Theory with existing algorithms. Establishing standardized modeling procedures and testing platforms can further facilitate the development process [3].

Subjectivity Issues: In certain applications, accurately determining the most likely, minimum, and maximum values of fuzzy numbers may introduce subjectivity, particularly in domains reliant on expert judgment. This can affect the objectivity of the final decision [7].

Solution: To reduce subjectivity errors, fuzzy number assessments can be combined with expert systems such as Analytical Hierarchy Process for weight learning [2]. Machine learning techniques can also be employed to automatically optimize fuzzy number parameters based on historical data.

7. Conclusion and Future Directions

Fermatean Fuzzy Theory has demonstrated strong potential for addressing uncertainty in AI decision-making, particularly in multi-criteria decision-making, compromise programming, and reinforcement learning. Despite persistent challenges, including computational complexity and sensitivity to data quality, its application in real-world settings indicates considerable robustness and adaptability. Future research should prioritize improving computational efficiency, enhancing data reliability, and extending its use to more complex decision-making environments. The proposed FF-DNN framework offers a promising pathway for uncertainty-aware AI algorithm optimization while avoiding substantial structural modifications to existing neural network architectures.

References

1. A. Kumari, D. Kumar, and K. Joshi, "An Application of Distance Measure Function of Fermatean Fuzzy Set in Urban Sustainable Development Appraisal," in *Advanced Technologies for Realizing Sustainable Development Goals: 5G, AI, Big Data, Blockchain, and Industry 4.0 Application*, Bentham Science Publishers, 2024, pp. 283-295.
2. M. Zhan and M. Zhang, "Fermatean Fuzzy TOPSIS Method Based on Prospect Theory and Its Application in Credit Assessment," *Procedia Computer Science*, vol. 242, pp. 928-935, 2024.
3. J. Cao, T. Zhou, S. Zhi, S. Lam, G. Ren, Y. Zhang, ... and J. Cai, "Fuzzy inference system with interpretable fuzzy rules: Advancing explainable artificial intelligence for disease diagnosis—A comprehensive review," *Information Sciences*, vol. 662, p. 120212, 2024.
4. K. C. Yuan, L. W. Tsai, K. H. Lee, Y. W. Cheng, S. C. Hsu, Y. S. Lo, and R. J. Chen, "The development an artificial intelligence algorithm for early sepsis diagnosis in the intensive care unit," *International Journal of Medical Informatics*, vol. 141, p. 104176, 2020.
5. Y. Li, "Research on neural network algorithm in artificial intelligence recognition," *Sustainable Energy Technologies and Assessments*, vol. 53, p. 102691, 2022.
6. A. Fernandez, F. Herrera, O. Cordon, M. J. del Jesus, and F. Marcelloni, "Evolutionary fuzzy systems for explainable artificial intelligence: Why, when, what for, and where to?," *IEEE Computational Intelligence Magazine*, vol. 14, no. 1, pp. 69-81, 2019.
7. B. Liu, *Uncertainty Theory: An Introduction to Its Axiomatic Foundation*. Heidelberg: Physica-Verlag, 2004.
8. T. M. Nishad, B. M. Harif, and A. Prasanna, "Decision support in legal systems through Fermat's fuzzy graph modeling," 2025.
9. H. J. Zimmermann, "Fuzzy set theory," *Wiley Interdisciplinary Reviews: Computational Statistics*, vol. 2, no. 3, pp. 317-332, 2010.

10. P. Vas, *Artificial-Intelligence-Based Electrical Machines and Drives: Application of Fuzzy, Neural, Fuzzy-Neural, and Genetic-Algorithm-Based Techniques*, vol. 45. Oxford: Oxford University Press, 1999.
11. A. Al Ka'bi, "Proposed artificial intelligence algorithm and deep learning techniques for development of higher education," *International Journal of Intelligent Networks*, vol. 4, pp. 68-73, 2023.
12. Y. Zheng, Z. Xu, and X. Wang, "The fusion of deep learning and fuzzy systems: A state-of-the-art survey," *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 8, pp. 2783-2799, 2021.